Strategic Technology Investment under Uncertainty

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In this paper the technology investment decision of a firm is analyzed, while competition on the output market is explicitly taken into account. Technology choice is irreversible and the firms face a stochastic innovation process with uncertainty about the speed of arrival of new technologies. The innovation process is exogenous to the firms. For reasons of market saturation and the fact that more modern technologies are invented as time passes, the demand for a given technology decreases over time. This implies that also the sunk cost investment of each technology decreases over time.

Introducing the waiting curve as a new concept, the investment decision problem is transformed into a timing game. An algorithm is designed for solving this (more) general timing game. The algorithm is applied to an information technology investment problem. The most likely outcome exhibits diffusion with equal payoffs for the firms.

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1. Introduction

In the last two decades information technology has become a very important determinant for economic growth[1]. For the individual firm the technology investment decision is a very complex matter due to the incredible rapid progress of information technology[2]. In former years technology investment decisions were mostly timing problems, in which the optimal time to replace the current technology had to be determined. As a result of the slow technological progress, only one new technology had to be taken into account. For example, at the beginning of this century the technology investment decision of a railway company dealt with the decision when to replace its steam shunters with diesel shunters. Up to the present day most railway companies still work with diesel shunters.

Nowadays, a firm should take into account that the current state of the art in information technology will be old fashioned in a few years. Thus the investment decision problem is no longer only a question of when to adopt a new technology but also a question of which technology should be adopted. Therefore it is important to design a theoretical framework in which several new technologies are considered. This paper wants to contribute to this

[1] Kriebel (1989) notes that roughly 50 procent of new corporate capital expenditures by major U.S. companies is in information technology.

[2] Yorukoglu (1998, p. 552): "Information technology capital has a very high pace of technological improvement. Compared with more traditional types of capital, the efficiency of information technology capital has increased much faster over the last few decades. As an example, consider the market for personal computers. IBM introduced its Pentium PCs in the early 1990s at the same price at which it introduced its 286 PCs in the 1980s. Therefore it took less than a decade for the computing technology to improve on the order of 20 times in terms of both speed and memory capacities, without increasing the cost."

aim.

Another feature of the last decade is that firms more and more face competition on their output markets. One reason is the abolition of monopolistic markets created by government. In the Netherlands examples are the opening of the markets for telecommunication, railway and power supply. Another reason is the, still ongoing, process of mergers, which due to legislation will not end with a market with only one supplier. The result is that markets with only one supplier and markets with many suppliers seem to disappear. Thus, in its own investment decision, a firm should take into account the investment behavior by its competitors, which is dealt with in this paper.

The existing literature on technology adoption models can be divided into two categories. The models in the first category are decision theoretic models that analyze the technology investment decision of a single firm. In the most advanced models there are multiple new technologies that arrive over time according to a stochastic process. Examples are Balcer and Lippman (1984), Nair (1995), Rajagopalan *et al.* (1998) and Farzin *et al.* (1998). The drawback of these models is that they analyze the investment decision of only one firm in isolation, so that the effects of competition are not incorporated.

The second category models are game theoretic models. Two (or more) firms compete on an output market and produce goods using a particular technology. Then, a new and more efficient technology is invented, and the question is at what time the firms should adopt it.

Reinganum (1981) was the first to analyze this kind of model. She considered a duopoly with identical firms, in which there is no uncertainty in the innovation process, and one new technology is considered. The investment expenditure required to adopt the new technology decreases over time and the efficiency improvement is known. If a firm adopts the new technology before the other one does, it makes substantial profits at the expense of the other firm. On the other hand the investment cost being decreasing over time provides an incentive to wait with investing. Reinganum assumes that the firms precommit themselves to adoption times, so she automatically obtains open-loop equilibria.

Fudenberg and Tirole (1985) proved that in these equilibria the leader (the firm that invests first) earns more than the follower (the firm that invests second). Since precommitment seems not to be very realistic in the strategic setting of a duopoly, Fudenberg and Tirole extend Reinganum's model by relaxing this assumption and by not determining beforehand which firm is the leader. They therefore allow preemption and obtain closed-loop equilibria with rent equalization.

The drawback of the Reinganum-Fudenberg-Tirole model is the assumption that there is only one new technology available for the firms, and, as stated in the beginning of this introduction, present technology investment decisions have to deal with a sequence of new technologies. In this paper we extend the Reinganum-Fudenberg-Tirole model by adding uncertainty to the innovation process and by considering several new technologies. The new technologies are invented at previously unknown points of time[3]. The investment decision problem is solved by introducing the waiting curve as a new concept in timing games.

The remainder of this paper is organized as follows. In Section 2 the investment decision problem of a firm is described. We reformulate the investment decision problem as a timing

[3] A different framework is considered in the duopoly model by Gaimon (1989). In that paper a continuous stream of new technologies arrives over time, which is known beforehand by the firms.

game, and design an algorithm to solve it in Section 3. In Section 4 we apply the algorithm to an information technology investment problem. Concluding remarks are given in Section 5.

2. Investment Decision Problem

In this section we describe the model of this paper. The first assumption is that firms are identical. Each firm has a profit function $\pi(\theta_x, \theta_y)$, where θ_x equals the technology-efficiency parameter of the technology that the firm uses itself and θ_y that of its opponent. The profit function of each firm is non-negative, increasing and concave in its own technology-efficiency parameter and decreasing in its rival technology-efficiency parameter. This for the reason that a firm can make more profits when it produces with a more efficient technology, the fact that the growth of the profits will be limited (due to output market saturation and the fact that production costs are always positive) and a firm will make less profits when its rival uses a more efficient technology:

$$\begin{aligned}
&\text{(i)} \quad \pi(\theta_x, \theta_y) \geq 0, \\
&\text{(ii)} \quad \tfrac{\partial \pi(\theta_x, \theta_y)}{\partial \theta_x} > 0, \\
&\text{(iii)} \quad \tfrac{\partial \pi(\theta_x, \theta_y)}{\partial \theta_y} < 0, \\
&\text{(iv)} \quad \tfrac{\partial^2 \pi(\theta_x, \theta_y)}{\partial \theta_x^2} < 0.
\end{aligned} \tag{1}$$

We analyze a dynamic model with an infinite planning horizon. Risk-neutral firms are considered, which discount the stream of future profits at a constant rate r. Initially each firm produces with a technology designated by $\theta = \theta_0$. As time passes new technologies become available (the i-th technology to arrive with efficiency θ_i), and each firm has the opportunity to adopt a new technology by investing $I(t)$, where t is the length of the time period passed since the introduction of that technology. We assume the second hand market for these capital goods to be negligible (e.g. information technology products) so that this investment is irreversible (this assumption is extensively motivated in Dixit and Pindyck (1994)). The differences between the technologies are all captured in the different values for θ, so that, without losing anything, investment expenditures ($= I(t)$) can be set equal for all technologies. The investment cost $I(\cdot)$ is non-negative, decreasing and convex in time:

$$\begin{aligned} &\text{(i)} \quad I(t) \geq 0, \\ &\text{(ii)} \quad \frac{\partial I(t)}{\partial t} \leq 0, \\ &\text{(iii)} \quad \frac{\partial^2 I(t)}{\partial t^2} \geq 0. \end{aligned} \tag{2}$$

Such a decrease can be motivated by the fact that better technologies become available as time passes so that the demand for the current technology decreases over time. Another factor can be learning by doing in the production process of the technology supplier.

Furthermore, we assume that the process of technological evolution (innovation supply) is exogenous to the firms. Technologies become more and more efficient over time, and the more efficient a technology the larger the associated parameter θ. However, the arrival process of the new technologies is a stochastic process. We assume that the associated increases in θ are known beforehand. In practice this occurs, for example, in the case of micro-chips where the technical parameters and specifications of future designs are known beforehand, but the arrival date is uncertain since the appearance of technology depends on research and development and market factors affecting the introduction of the product (Nair (1995)).

We denote the newest or best technology available at time t by $B(t)$. To incorporate the uncertainty in the innovation process we assume that $B(t)$ is a Poisson process with rate λ. We define T_i to be equal to the point in time at which technology i becomes available. The new technology i has efficiency level equal to $\theta_i > \theta_{i-1}$, $i = 1, 2, \ldots$. The interarrival time τ_i is the time between the invention times of the $(i+1)$-th and i-th technology:

$$\tau_i = T_{i+1} - T_i, \quad i = 1, 2, \ldots. \tag{3}$$

As a result of the Poisson arrival process the τ_i's are independently and identically distributed according to an exponential distribution with parameter λ.

3. Timing Game

For simplicity reasons we restrict ourselves to the case where firms can only make one technology switch. This more or less holds for firms whose financial means are limited. We transform the investment decision problem into a two player timing game.

In a timing game each player has to decide when to make a single move. The player that moves first is called the leader and the other is the follower. Since firms are identical there seems to be no reason why one of these firms should be given the leader role beforehand. Therefore, we strive at obtaining equilibria where it is not known beforehand which firm will invest first. In the general setting of a timing game the payoff of a player depends on its own date of moving and the other player's date of moving. In case one player has already moved, the problem for the other player is a one person decision problem. A player can react instantaneously to its opponent's action.

Four payoff curves are important in our timing game. Each payoff curve is a function of time t, which is the time passed since the last technology has become available for the firms. Let the leader move at time t. Then the value of the follower, which is the outcome of the one person's decision problem, is denoted by $F(t)$. The value of the leader is given by $L(t)$, in which the optimal action of the follower is included. In case of a simultaneous move at time t the value of a player is denoted by $M(t)$. Since simultaneous moving is always possible for the follower, it holds that

$$F(t) \geq M(t), \text{ for all } t. \tag{4}$$

The fourth curve is called the waiting curve, which is a new concept within the area of timing games. Here, the waiting curve is used to transform the investment decision problem under consideration into a timing game. The waiting curve represents the expected payoff of a firm if both firms do not move (at least) until the next arrival of a new technology and act optimally afterwards. This implies that we need to know the equilibrium outcome of the game that starts after the arrival of the next technology. As a result we have to consider a finite number of new technologies. This assumption is not too strict due to discounting. In order to find the right number of new technologies to take into account in the model, the following algorithm can be used:

Step 0 Solve the model with one technology.

Step 1 Add one extra technology to the model and solve the model.

Step 2 If the results of the last two models are very different go to step 1, otherwise the right number of technologies has been found.

A model with n new technologies is solved as follows. Start with solving the timing game that starts after the arrival of the n-th technology. This game is a classical timing game, since it contains no waiting curve. The equilibrium outcomes of this game are used to construct the waiting curve for the game that starts at some time during the interval $[T_{n-1}, T_n)$. Solve this game and use the equilibrium outcomes to construct the waiting curve for the game that starts somewhere at the time interval $[T_{n-2}, T_{n-1})$. This procedure goes on until the game that starts at time $T_1 = 0$ is solved.

This section describes the construction of the four payoff curves. In Subsection 3.1 we derive the value of a firm given each firm's strategy. After that we determine the leader, follower and joint-moving curves using this value function in Subsection 3.2. In Subsection 3.3 possible equilibria of timing games without waiting curve are explained. The waiting curve is constructed in Subsection 3.4. In that subsection we also explain the implication of adding the waiting curve for the possible equilibria of timing games. Finally, in Subsection 3.5 the algorithm for solving the investment decision problem with a finite number of new technologies is summarized.

3.1. Value Function

In this investment decision problem firms not only have to decide when to adopt a technology, but also which technology to adopt. Define $V(s, i, t, j)$ as the expected value (at time 0) of a firm that adopts technology i itself at time $T_i + s$, [4] while its rival adopts technology j at time $T_j + t$. If (in expectation) the firm adopts as second, thus $T_j + t \leq T_i + s$,[5] the

[4] Note that we denote the stochastic variable describing the arrival date of technology i and the realization of that stochastic variable by the same symbol T_i.

[5] If T_i and T_j are both stochastic we define that it holds that $T_i \geq T_j$, if and only if $E[T_i] \geq E[T_j]$.

expected value of the firm equals

$$V(s,i,t,j) = E\left[\int_{u=0}^{T_j+t} \pi(\theta_0,\theta_0)\, e^{-ru} du + \int_{u=T_j+t}^{T_i+s} \pi(\theta_0,\theta_j)\, e^{-ru} du \right. \tag{5}$$
$$\left. + \int_{u=T_i+s}^{\infty} \pi(\theta_i,\theta_j)\, e^{-ru} du - I(s)\, e^{-r(T_i+s)}\right],$$

and if $T_j + t > T_i + s$:

$$V(s,i,t,j) = E\left[\int_{u=0}^{T_i+s} \pi(\theta_0,\theta_0)\, e^{-ru} du + \int_{u=T_i+s}^{T_j+t} \pi(\theta_i,\theta_0)\, e^{-ru} du \right. \tag{6}$$
$$\left. + \int_{u=T_j+t}^{\infty} \pi(\theta_i,\theta_j)\, e^{-ru} du - I(s)\, e^{-r(T_i+s)}\right].$$

Rewriting (5) gives

$$\begin{aligned} V(s,i,t,j) = &\frac{\pi(\theta_0,\theta_0)}{r}\left(1 - E\left[e^{-r(T_j+t)}\right]\right) \\ &+\frac{\pi(\theta_0,\theta_j)}{r}\left(E\left[e^{-r(T_j+t)}\right] - E\left[e^{-r(T_i+s)}\right]\right) \\ &+\frac{\pi(\theta_i,\theta_j)}{r} E\left[e^{-r(T_i+s)}\right] - I(s)\, E\left[e^{-r(T_i+s)}\right]. \end{aligned} \tag{7}$$

Equation (6) can be written as follows

$$\begin{aligned} V(s,i,t,j) = &\frac{\pi(\theta_0,\theta_0)}{r}\left(1 - E\left[e^{-r(T_i+s)}\right]\right) \\ &+\frac{\pi(\theta_i,\theta_0)}{r}\left(E\left[e^{-r(T_i+s)}\right] - E\left[e^{-r(T_j+t)}\right]\right) \\ &+\frac{\pi(\theta_i,\theta_j)}{r} E\left[e^{-r(T_j+t)}\right] - I(s)\, E\left[e^{-r(T_i+s)}\right]. \end{aligned} \tag{8}$$

Since the values for s and t are determined by the firms themselves, they are known for sure. This implies that for determining these value functions there is one thing left to derive: an expression for $E\left[e^{-rT_k}\right]$, where $k \in \{i,j\}$. It is obvious that if technology k has arrived before some time T, its arrival date T_k is known for sure at this time T. Now consider the situation that technology k has not been invented by time T. Please remember that $B(T)$ is the number of the most efficient technology that is available at time T. We denote the number of technologies that arrive during an interval $[T, T+S)$ by $R(S)$. Thus, it holds that $R(S) = B(T+S) - B(T)$. Due to the fact that B is a Poisson process with rate λ, the stochastic variable $R(S)$ is distributed according to a Poisson distribution with parameter λS. Now it is not hard to see that

$$\Pr\left(T_k - T \leq S \middle| B(T) < k\right) = \sum_{i=k-B(T)}^{\infty} \Pr\left(R(S) = i\right) \tag{9}$$

Using equation (9) we derive the following expression for $E\left[e^{-r(T_k-T)} \middle| B(T) < k\right]$ [6]:

$$\begin{aligned}
E\left[e^{-r(T_k-T)} \middle| B(T) < k\right] &= 1 - r \int_{u=0}^{\infty} e^{-ru}\left(1 - \Pr\left(T_k - T \leq u\right)\right) du \\
&= 1 - r \int_{u=0}^{\infty} e^{-ru}\left(1 - \sum_{i=k-B(T)}^{\infty} \Pr\left(R(u) = i\right)\right) du \\
&= 1 - r \int_{u=0}^{\infty} e^{-ru} \sum_{i=0}^{k-B(T)-1} \Pr\left(R(u) = i\right) du \\
&= 1 - r \sum_{i=0}^{k-B(T)-1} \int_{u=0}^{\infty} e^{-ru} e^{-\lambda u} \frac{(\lambda u)^i}{i!} du
\end{aligned}$$

[6] Assume that the stochastic variable X is distributed over the interval $[0, \infty)$ according to some distribution with distribution function $F(x) := \Pr(X \leq x)$, and that the function $f(x)$ is continuous and differentiable in x.

Then $E[f(X)] = \int_{x=0}^{\infty} f(x)\, dF(x) = \int_{x=0}^{\infty} \left[\int_{t=0}^{x} f'(t)\, dt + f(0)\right] dF(x) = \int_{t=0}^{\infty} \left[\int_{x=t}^{\infty} f'(t)\, dF(x)\right] dt + f(0) \int_{x=0}^{\infty} dF(x) = \int_{t=0}^{\infty} f'(t)(1 - F(t))\, dt + f(0)$.

$$
\begin{aligned}
&= 1 - r \sum_{i=0}^{k-B(T)-1} \frac{\lambda^i}{(r+\lambda)^{i+1}} \int_{u=0}^{\infty} \frac{(r+\lambda)^{i+1} e^{-(r+\lambda)u} u^i}{i!} du \\
&= 1 - \frac{r}{(r+\lambda)} \frac{1 - \left(\frac{\lambda}{r+\lambda}\right)^{k-B(T)}}{1 - \left(\frac{\lambda}{r+\lambda}\right)} \\
&= \left(\frac{\lambda}{r+\lambda}\right)^{k-B(T)}. \qquad (10)
\end{aligned}
$$

With the help of equation (10) we derive that

$$
E_T\left[e^{-rT_k}\right] = \begin{cases} e^{-rT}\left(\frac{\lambda}{r+\lambda}\right)^{k-B(T)} & B(T) < k, \\ e^{-rT_k} & B(T) \geq k. \end{cases} \qquad (11)
$$

3.2. Leader, Follower and Joint-Moving Curves

At each point of time t the leader can choose to immediately invest in a technology j from the finite set $\{1, 2, \ldots, B(t)\}$. Given an adoption strategy of the leader (t, j) the optimal reaction of the follower can be calculated in two steps.

In the first step, derive for each technology i the optimal adoption date s_i^* for the follower. Since the follower's payoff depends on the technology the other firm uses, s_i^* will be a function of t and j. Therefore,

$$
s_i^*(t, j) = \arg \max_{u \geq \max(T_i, t)} V(u, i, t, j). \qquad (12)
$$

In order to be more specific about $s_i^*(t, j)$ consider the following scenario: the leader has already adopted technology j and technology i has just been invented. The follower can either adopt technology i right away or delay adoption. Let $w_i^F(j)$ denote the optimal waiting time for the follower, that is the length of the time period between invention and optimal adoption of technology i. Solving the maximization problem (12), in which $V(\cdot,\cdot,\cdot,\cdot)$ is given by (7), yields that $w_i^F(j)=0$ if

$$\pi(\theta_i,\theta_j)-\pi(\theta_0,\theta_j)\geq rI(0)-I'(0), \tag{13}$$

and that $w_i^F(j)$ is implicitly determined by

$$\pi(\theta_i,\theta_j)-\pi(\theta_0,\theta_j)-rI\left(w_i^F(j)\right)+I'\left(w_i^F(j)\right)=0, \tag{14}$$

otherwise. Equation (14) states that the marginal costs, $rI\left(w_i^F(j)\right)-I'\left(w_i^F(j)\right)$, and the marginal benefits, $\pi(\theta_i,\theta_j)-\pi(\theta_0,\theta_j)$, are balanced at time $w_i^F(j)$. The marginal costs are equal to the opportunity costs of the investment, $rI(t)$, and the costs resulting from the fact that the firm invests right away so that it does not take advantage from I being decreasing over time. If at time 0 the marginal benefits exceed the marginal costs (cf. equation (13)) the firm should adopt immediately so that $w_i^F(j)=0$.

Using the definition of $w_i^F(j)$[7], we extend this particular scenario to the general case and conclude that the optimal adoption time $s_i^*(t,j)$ is equal to

$$s_i^*(t,j)=\begin{cases} t & \text{if } t\geq T_i+w_i^F(j), \\ T_i+w_i^F(j) & \text{if } t<T_i+w_i^F(j). \end{cases} \tag{15}$$

In the second step, use (15) to determine the technology i^* that maximizes the follower's payoff, given that the leader invests at time t in technology j:

$$i^*(t,j)=\arg\max_k V\left(s_k^*(t,j),k,t,j\right). \tag{16}$$

[7] Since $\pi(\theta_i,\theta_j)$ is increasing in θ_i , we know that there exists a $\widehat{\imath}(j)$ such that $w_i^F(j)=0$ for all $i\geq\widehat{\imath}(j)$.

The leader, having read the previous lines, takes into account the follower's investment behavior in choosing at time t the technology $j^*(t)$ that results in the largest payoff:

$$j^*(t) = \arg \max_{k\in\{1,2,\ldots,B(t)\}} V\left(t, k, s^*_{i^*(t,k)}(t,k), i^*(t,k)\right). \tag{17}$$

The process described above results in the following value functions for the timing game:

$$L(t) = V\left(t, j^*(t), s^*_{i^*(t,j^*(t))}(t, j^*(t)), i^*(t, j^*(t))\right), \tag{18}$$

$$F(t) = V\left(s^*_{i^*(t,j^*(t))}(t, j^*(t)), i^*(t, j^*(t)), t, j^*(t)\right), \tag{19}$$

$$M(t) = V(t, j^*(t), t, j^*(t)). \tag{20}$$

3.3. Equilibria for Timing Games without Waiting Curve

In this subsection possible equilibria for classical timing games, i.e. timing games without waiting curves, are presented. In our model with n new technologies the game that starts after time T_n is a classical timing game.

Classical timing games can be divided in two classes. The first class consists of the so-called preemption games and the elements of the second class are called wars of attrition. Preemption games are characterized by the fact that there exists a point of time where there is a first mover advantage:

$$\exists t \in [0,\infty) \text{ such that } L(t) > F(t). \tag{21}$$

In a war of attrition the follower's payoff exceeds the leader's payoff at all times:

$$F(t) > L(t) \text{ for all } t \in [0,\infty). \tag{22}$$

In general a (classical) timing game can be split up into a finite number of subgames, where each subgame is a preemption game or a war of attrition. Due to the definitions of preemption games and wars of attrition, the split up points will be the points at which the function $L(t) - F(t)$ changes its sign. The equilibrium of a general timing game is found by first solving the last subgame, using the resulting value functions of the equilibrium of this subgame in the second last subgame and so forth and so on.

Since we analyze identical firms we are especially interested in equilibria with symmetric strategies. For identical and rational firms there is no reason why they should act differently. An example and a rigorous treatment of a preemption game can be found in Fudenberg and Tirole (1985). Hendricks *et al.* (1988) analyze wars of attrition in detail and equilibrium strategies can be found in that paper.

The equilibrium outcome of the timing game that starts after time T_n depends on the interarrival time τ_{n-1}. We denote the (expected) equilibrium outcome of the game that starts after time T_n by $\Omega_n(\tau_{n-1})$. If the game has more than one equilibrium, we use the most reasonable[8] equilibrium in the calculations.

3.4. Waiting Curve

The waiting curve for the game that starts somewhere during the time interval $[T_{n-1}, T_n)$ equals:

$$\begin{aligned} W(t) = &\int_{v=0}^{t} \pi(\theta_0, \theta_0) e^{-rv} dv \\ &+ e^{-rt} \int_{\tau_{n-1}=0}^{\infty} \left[\int_{u=0}^{\tau_{n-1}} \pi(\theta_0, \theta_0) e^{-ru} du + e^{-r\tau_{n-1}} \Omega_n(\tau_{n-1}) \right] \lambda e^{-\lambda \tau_{n-1}} d\tau_{n-1}. \end{aligned} \tag{23}$$

[8] The most reasonable equilibrium is defined as the equilibrium under which the player's payoffs are maximal (the Pareto optimal equilibrium, cf. Fudenberg and Tirole (1985))

The first part represents the profits made by the firm on the time interval $[0, t]$. The second part resembles the expected payoff of the firm from time t onwards conditioned on the interarrival time τ_{n-1}, where $\Omega_n(\tau_{n-1})$ represents the (expected) outcome of the game that arises at the moment that the n-th technology is invented.

Consider a timing game for which the leader payoff exceeds the waiting payoff for some time t: $L(t) > W(t)$. Then at least one firm is better off by investing at time t. Thus a waiting curve in a timing game only matters if it holds that $W(t) > L(t)$ for all $t \in [0, \infty)$. In such a game the equilibrium strategy for both firms is to refrain from investment at least until the arrival of the next technology. Thus the equilibrium outcome of a game with waiting curve equals the equilibrium outcome of the same game without waiting curve if it holds that there exists a time t for which $L(t) > W(t)$. Otherwise, the value of the equilibrium outcome at some point in time is equal to the waiting curve at that time instant.

In general, the equilibrium outcome of the game that starts in the interval $[T_k, T_{k+1})$ is denoted by $\Omega_k(\tau_{k-1})$. Using this notation, the general waiting curve for a game that starts in the interval $[T_{k-1}, T_k)$ equals

$$W(t) = \int_{v=0}^{t} \pi(\theta_0, \theta_0) e^{-rv} dv \\ + e^{-rt} \int_{\tau_{k-1}=0}^{\infty} \left[\int_{u=0}^{\tau_{k-1}} \pi(\theta_0, \theta_0) e^{-ru} du + e^{-r\tau_{k-1}} \Omega_k(\tau_{k-1}) \right] \lambda e^{-\lambda \tau_{k-1}} d\tau_{k-1}. \tag{24}$$

3.5. Solution Procedure

In this subsection the solution procedure is summarized. In the first step the classical timing game that starts at time T_n is solved. This gives the equilibrium outcome function $\Omega_n(\tau_{n-1})$. Using this equilibrium outcome function we construct the waiting curve (23) and solve the timing game that starts at a point in time on the interval $[T_{n-1}, T_n)$. The resulting outcomes are incorporated in the function $\Omega_{n-1}(\tau_{n-2})$ which is again used to construct the waiting curve for the timing game that starts somewhere during the time interval $[T_{n-2}, T_{n-1})$. This process is repeated until the game that starts at T_1 is solved.

Combining the equilibrium strategies of each step gives the optimal investment strategy of the firm. The ex-ante probabilities of each equilibrium outcome can be derived using the calculations of each step. After each realization of an interarrival time these probabilities must be updated.

4. The Information Technology Investment Problem

In this section we apply the algorithm of the previous section to a specific information technology investment problem. Information technology products are heavily dependent on micro-chips. The memory and arithmetic power of micro-chips develop in an exponential way over time. This was firstly recognized by one of the Intel-founders Gordon Moore in 1964, who found out that the amount of information on a piece of silicium doubles every year. This statement is called Moore's law. Nowadays, Moore's law still applies although the doubling time has risen to two to three years. In our calculations it is assumed that on average every three years a new generation of chips arrives: $\lambda = \frac{1}{3}$. A new generation of chips is a generation that is twice as efficient as the preceding generation. After applying the algorithm stated in the beginning of Section 3, it turned out that we need to take four generations of chips into account. After normalizing the technology parameter of the current technology to one, this gives rise to the following θ scheme:

$$\theta_0 = 1, \tag{25}$$

$$\theta_{i+1} = 2\theta_i, \; i \in \{0, 1, 2, 3\}, \tag{26}$$

so that

$$\theta_i = 2^i, \; i \in \{0, 1, 2, 3, 4\}. \tag{27}$$

Due to the rapid innovation process, prices of information technology products go down quickly. Therefore, we assume that

$$I(t) = I_0 \exp(-\alpha t), \tag{28}$$

where

$$I_0 = 50, \tag{29}$$

$$\alpha = 1. \tag{30}$$

Formulas (28)-(30) hold for every technology we consider. Hence, the technologies only differ in their technology parameter. The strange thing with micro-electronics is that their fast efficiency improvement does not impress consumers. As an illustration, consider a telephone in which a certain amount of telephone numbers can be stored. A new generation of chips doubles this amount, but most likely this will not be a reason for the customers to sell their old telephone and buy a new one. Another example is that a new generation of personal computers will not double the research output of a scientist. Therefore, a manager of Philips, Theo Claassen, has argued that utility is a logarithmic function of technology, in the sense

τ_3 region	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
$\tau_3 \in [0, 0.800591)$	P	4	$T_4 + t_4^P$	4	$T_4 + w_4^F(4)$
$\tau_3 \in [0.800591, 1.17938)$	WA	3	$T_4 + t_{34}^L(\tau_3)$	4	$T_4 + w_4^F(3)$
$\tau_3 \in [1.17938, 1.87931]$	WA	3	$T_4 + S_{34}^L(\tau_3)$	4	$T_4 + w_4^F(3)$
$\tau_3 \in (1.87931, 1.89322)$	P	3	$T_4 + t_{34}^P(\tau_3)$	4	$T_4 + w_4^F(3)$
$\tau_3 \in [1.89322, \infty)$	P	3	T_4	4	$T_4 + w_4^F(3)$

Table 1. Equilibria and type of subgames starting at time T_4 as function of τ_3. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption times are defined as follows: $t_4^P := \min\left\{t \left| V\left(t, 4, w_4^F(4), 4\right) = V\left(w_4^F(4), 4, t, 4\right)\right.\right\} = 0.734579$, $t_{34}^L(\tau_3) := \min\left\{t \left| V\left(t, 3, w_4^F(3), 4\right) = V\left(t, 4, w_4^F(4), 4\right)\right.\right\}$, $S_{34}^L(\tau_3) := \arg\max_{t \in [0, t_{34}^L(\tau_3)]} V\left(t, 3, w_4^F(3), 4\right)$ and $t_{34}^P(\tau_3) := \min\left\{t \left| V\left(t, 3, w_4^F(3), 4\right) = V\left(w_4^F(3), 4, t, 4\right)\right.\right\}$.

that utility doubles in case technology power becomes ten times as large[9]. For this reason we assume that profit increases with the technology-efficiency parameter in a logarithmic way with base 10 (cf. (27)):

$$\pi(\theta_i, \theta_j) = \frac{{}^{10}\log\left(2\theta_i^2\right)}{{}^{10}\log(2\theta_j)} = \frac{2i+1}{j+1}. \tag{31}$$

The discount rate equals $r = 0.05$. From equations (13), (14), (28) and (31) we derive that

$$w_i^F(j) = \begin{cases} \frac{1}{\alpha}\log\left(\frac{(r+\alpha)I_0(j+1)}{2i}\right) & \text{if } i < \frac{1}{2}(j+1)(r+\alpha)I_0, \\ 0 & \text{else.} \end{cases} \tag{32}$$

In Appendix A the expected equilibrium outcomes for the subgames starting right at the invention times T_4, T_3 and T_2 are derived. The results are summarized in Tables 1-3.

In Tables 2 and 3 the equilibrium outcomes are conditional on the next technology not arriving too early. That is the next technology does not arrive before the time at which the

[9] This was stated in the Dutch magazine Elsevier (January 24, 1998).

τ_2 region	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
$[0, 1.24843)$	P	3	$T_3 + t_{34}^P$	4	$T_4 + w_4^F(3)$
$[1.24843, 2.94586)$	WA	2	$T_3 + t_{23}^L(\tau_2)$	4	$T_4 + w_4^F(2)$
$[2.94586, 3.95758)$	WA	2	$T_3 + S_{24}^L(\tau_2)$	4	$T_4 + w_4^F(2)$
$[3.95758, \infty)$	WA	2	T_3	4	$T_4 + w_4^F(2)$

Table 2. Equilibria and type of subgames starting at time T_3 as function of τ_2. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption times are defined as follows: $t_{34}^P := \min\left\{t \,\middle|\, V\left(t, 3, w_4^F(3), 4\right) = V\left(w_4^F(3), 4, t, 3\right)\right\} = 0.727495$, $t_{23}^L(\tau_2) := \min\left\{t \,\middle|\, V\left(t, 2, w_4^F(2), 4\right) = V\left(t, 3, w_4^F(3), 4\right)\right\}$ and $S_{24}^L(\tau_2) := \arg\max_{t\in[0, t_{24}^L(\tau_2)]} V\left(t, 2, w_4^F(2), 4\right)$.

τ_1 region	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
$[0, \infty)$	P	2	$T_2 + t_{24}^P$	4	$T_4 + w_4^F(2)$

Table 3. Equilibria and type of subgames starting at time T_2 as function of τ_1. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption time is defined as follows: $t_{24}^P := \min\left\{t \,\middle|\, V\left(t, 2, w_4^F(2), 4\right) = V\left(w_4^F(2), 4, t, 2\right)\right\} = 1.81706$.

leader changes technologies according to the table. In Appendix A the equilibrium outcome functions $\Omega_i(\tau_{i-1})$, $i = 2, 3, 4$, are derived from Tables 1-3.

If technology 4 arrives shortly after technology 3 (see first line of Table 1), technology 4 dominates technology 3 and both firms will adopt technology 4. If it takes a little longer before technology 4 becomes available, technology 3 is the most attractive technology for the leader to adopt. In the second and third τ_3 region the follower's value is higher than the leader's value. To explain this second mover advantage, consider the second line of Table 1. The value of the gain of market share of the follower during the time interval $\left[T_4 + w_4^F(3), \infty\right)$ outweighs the value of the gain of market share of the leader during the interval $\left[T_4 + t_{34}^L(\tau_3), T_4 + w_4^F(3)\right)$. A late arrival of technology 4 makes technology 3 attractive enough for direct adoption, see last line of Table 1. Tables 2-3 should be interpreted in the same way.

We now analyze the game at the moment where technologies 2, 3 and 4 are not invented yet in a more elaborate way. Using the outcome function $\Omega_2(\tau_1)$ we construct the waiting curve for the game that starts at time T_1 (cf. (24)), which is the invention time of the first technology:

$$W(t) = \frac{\pi(\theta_0, \theta_0)}{r}\left(1 - e^{-rt}\right) + \int_{\tau_1=0}^{\infty}\left[\int_{u=0}^{\tau_1} \pi(\theta_0, \theta_0) e^{-ru} du + e^{-r\tau_1}\Omega_2(\tau_1)\right] \lambda e^{-\lambda\tau_1} d\tau_1. \tag{33}$$

The leader, follower and joint-moving curves are derived with the equations presented in Section 3. In Figure 1 the four curves are plotted.

From Figure 1 the following ordering of the curves is derived: $F(t) > W(t) > L(t) > M(t)$ for all $t \in [T_1, T_2)$. This implies that each firm likes the other to invest as first and does not want to invest as first itself. Thus waiting is the optimal strategy for the firms in the game that starts in the interval $[T_1, T_2)$.

Then at time T_2 the game starts where technologies 1 and 2 are present, but the remaining technologies 3 and 4 are not invented yet. From Table 3 we derive that one firm will adopt technology 2 at time $T_2 + t_{24}^P$ and the other technology 4 if the third technology does not arrive before time $T_2 + t_{24}^P$. With probability

$$\Pr\left(\tau_2 \geq t_{24}^P\right) = e^{-\lambda t_{24}^P} = 0.54570,$$

this is the case.

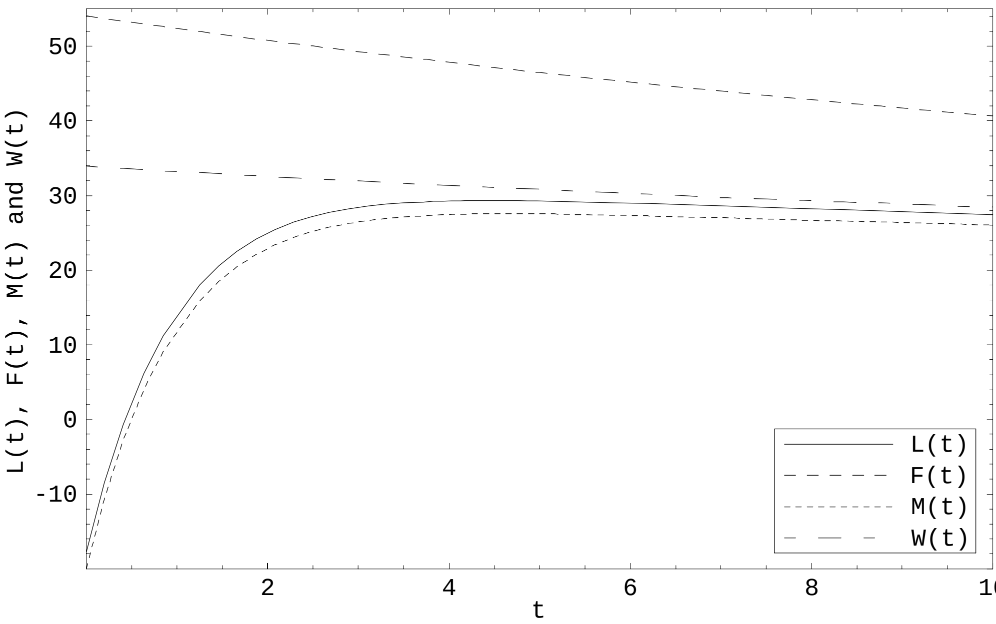

Figure 1. $T_1 = 0$.

With probability

$$\Pr\left(\tau_2 < t_{24}^P\right) = 1 - e^{-\lambda t_{24}^P} = 0.45430,$$

technology 3 arrives before time $T_2 + t_{24}^P$. Now, there are two cases. In case A, τ_2 is smaller than the boundary 1.24843 (see Table 2), which occurs with probability

$$\Pr\left(\tau_2 < 1.24843\right) = 1 - e^{-\lambda \cdot 1.24843} = 0.34041,$$

and in case B, $1.24843 < \tau_2 < t_{24}^P$, which occurs with probability

$$\Pr\left(1.24843 \leq \tau_2 < t_{24}^P\right) = e^{-\lambda \cdot 1.24843} - e^{-\lambda t_{24}^P} = 0.11389.$$

Table 2 states that, in case A, the outcome will be adoption of technology 3 at time $T_3 + t_{34}^P$ if technology 4 does not arrive before that time. This outcome occurs with the following probability:

$$\begin{aligned}
\Pr\left(\tau_2 < t_{24}^P \text{ and } \tau_3 \geq t_{34}^P\right) &= \Pr\left(\tau_2 < t_{24}^P\right) \Pr\left(\tau_3 \geq t_{34}^P\right) \\
&= \left(1 - e^{-\lambda t_{24}^P}\right) e^{-\lambda t_{34}^P} \\
&= 0.26711.
\end{aligned}$$

Technology 4 arrives before time $T_3 + t_{34}^P$, with probability

$$\begin{aligned}
\Pr\left(\tau_2 < t_{24}^P \text{ and } \tau_3 < t_{34}^P\right) &= \Pr\left(\tau_2 < t_{24}^P\right) \Pr\left(\tau_3 < t_{34}^P\right) \\
&= \left(1 - e^{-\lambda t_{24}^P}\right) \left(1 - e^{-\lambda t_{34}^P}\right) \\
&= 0.073303.
\end{aligned}$$

In this case the outcome will be a preemption equilibrium in which one firm adopts technology 4 at time $T_4 + t_4^P$ and the other firm technology 4 at time $T_4 + w_4^F(4)$. Here it is important to note that $t_{34}^P = 0.727495$ is smaller than the first τ_3 boundary 0.800591. Hence, with probability one the outcomes listed on the lines 2-5 of Table 1 will not occur here.

Case B is a little more complicated. The outcome exhibits adoption of technology 3 at time $T_3 + t_{23}^L(\tau_2)$ by one firm (the other firm adopts technology 4) if technology 4 arrives after time $T_3 + t_{23}^L(\tau_2)$, which happens with probability:

$$\begin{aligned}
\Pr\left(1.24843 \leq \tau_2 < t_{24}^P \text{ and } \tau_3 \geq t_{34}^L(\tau_2)\right) &= \int_{\tau_2=1.24843}^{t_{24}^P} \Pr\left(\tau_3 \geq t_{34}^L(\tau_2)\right) \lambda e^{-\lambda \tau_2} d\tau_2 \\
&= \int_{\tau_2=1.24843}^{t_{24}^P} e^{-\lambda t_{34}^L(\tau_2)} \lambda e^{-\lambda \tau_2} d\tau_2 \\
&= 0.086802.
\end{aligned}$$

Otherwise the outcome is of the preemption type (first line of Table 1) if $\tau_3 < 0.800591$ or a war of attrition (second line of Table 1) if $\tau_3 \geq 0.800591$. The probability that the preemption equilibrium occurs is equal to

$$\begin{aligned} & \Pr\left(1.24843 \leq \tau_2 < t_{24}^P \text{ and } \tau_3 < t_{34}^L(\tau_2) \text{ and } \tau_3 < 0.800591\right) \\ = & \int_{\tau_2=1.24843}^{t_{24}^P} \Pr\left(\tau_3 < \min\left(t_{34}^L(\tau_2), 0.800591\right)\right) \lambda e^{-\lambda\tau_2} d\tau_2 \\ = & \, 0.026273. \end{aligned}$$

With probability

$$\begin{aligned} & \Pr\left(1.24843 \leq \tau_2 < t_{24}^P \text{ and } \tau_3 < t_{34}^L(\tau_2) \text{ and } \tau_3 \geq 0.800591\right) \\ = & \int_{\tau_2=1.24843}^{t_{24}^P} \Pr\left(0.800591 \leq \tau_3 < t_{34}^L(\tau_2)\right) \lambda e^{-\lambda\tau_2} d\tau_2 \\ = & \int_{\tau_2=1.24843}^{t_{24}^P} \left(e^{-\lambda \cdot 0.800591} - e^{-\lambda t_{34}^L(\tau_2)}\right) 1_{\left\{0.800591 \leq t_{34}^L(\tau_2)\right\}} \lambda e^{-\lambda\tau_2} d\tau_2 \\ = & \, 0.00081337, \end{aligned}$$

the war of attrition will happen. Here the leader adopts technology 3 and the follower invests in technology 4. So, on the longer term the follower produces with the more efficient technology which here leads to a higher payoff.

The analysis above implies that only the first two lines of Tables 1 and 2 matter. This for the reason that one of the firms adopts an existing technology, if a new technology arrives too late.

In Table 4 all possible outcomes and the probabilities are summarized. We conclude that the ex-ante probability of a preemption equilibrium with rent equalization (see Appendix

Probability	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
0.54570	P	2	$T_2 + t_{24}^P$	4	$T_4 + w_4^F(2)$
0.26711	P	3	$T_3 + t_{34}^P$	4	$T_4 + w_4^F(3)$
0.086802	WA	2	$T_3 + t_{23}^L(\tau_2)$	4	$T_4 + w_4^F(2)$
0.099576	P	4	$T_4 + t_4^P$	4	$T_4 + w_4^F(4)$
0.00081337	WA	3	$T_4 + t_{34}^L(\tau_3)$	4	$T_4 + w_4^F(3)$

Table 4. Equilibria and ex-ante probabilities at time $T_1 = 0$. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption times are defined as follows: $t_{24}^P := \min\left\{t \left| V\left(t, 2, w_4^F(2), 4\right) = V\left(w_4^F(2), 4, t, 2\right)\right.\right\} = 1.81706$, $t_{34}^P := \min\left\{t \left| V\left(t, 3, w_4^F(3), 4\right) = V\left(w_4^F(3), 4, t, 3\right)\right.\right\} = 0.727495$, $t_{23}^L(\tau_2) := \min\left\{t \left| V\left(t, 2, w_4^F(2), 4\right) = V\left(t, 3, w_4^F(3), 4\right)\right.\right\}$, $t_4^P := \min\left\{t \left| V\left(t, 4, w_4^F(4), 4\right) = V\left(w_4^F(4), 4, t, 4\right)\right.\right\} = 0.734579$ and $t_{34}^L(\tau_3) := \min\left\{t \left| V\left(t, 3, w_4^F(3), 4\right) = V\left(t, 4, w_4^F(4), 4\right)\right.\right\}$.

A) equals 0.91238. The most likely outcome (probability 0.54570) is that one firm adopts technology 2 and the other firm technology 4. With probability 0.087615 there is a second mover advantage in the equilibrium, i.e. the firm that invests as first earns less than the firm that invests as second. The market share gain by the second mover offsets the temporary market share gain of the first mover. With probability 0.90042 the leader adopts another technology than the follower. The follower is expected to adopt technology 4 in all equilibria. Joint adoption does not occur as an equilibrium outcome.

We did not add an extra new technology to the model, because the probability that both firms adopt technology 4 is less than 0.10. Hence, with a probability of more than 90% one firm invests in another technology than the last one. For this reason we choose not to analyze the game with one technology more.

5. Conclusion

We analyzed a framework in which consecutive generations of new technologies arrive over time, and a firm has to make its optimal technology investment decision. Competition on the output market is taken into account. As time passes more efficient technologies arrive according to a stochastic arrival process. The investment cost of a particular technology drops over time.

Introducing the waiting curve as a new concept, the investment decision problem was converted into a timing game. The timing game changes every time a new technology enters the market. We designed an algorithm that can be used to solve this game.

The algorithm is applied to an information technology investment problem with four new technologies. The most likely outcome exhibits diffusion, one firm adopts technology 2 early and the other technology 4 later on, while the expected payoffs of the first and second investor are the same. With a probability of more than 90% the expected payoffs of the firms are equal. In the other cases the firm that invests as second does better than the firm that invests as first. Thus the temporary gain of market share by the leader does not make up for the market share gain of the follower.

One possible extension of this model is to relax the assumption that firms are allowed to make only one technology switch. We believe that this model can be solved in the same fashion: use the waiting curve concept to convert the game to a timing game with multiple actions and solve that game following the work by Simon (1987).

Another interesting extension is to make the number of active firms on the output market endogenous. If the active firms make positive profits it may be interesting for a new firm to enter the market. How does the threat of entering change the technology adoption behavior of the existing firms? Will they try to prevent firms to enter the market by adopting new technologies sooner?

Appendix

A. Construction of the Waiting Curve

Here the waiting curve is constructed. To do so, starting out from each realization the subgames have to be solved. Appendix B provides some relevant mathematical prerequisites for the analysis in Appendix A.

A.1. Games Starting at Time $t \in [T_4, \infty)$

The outcome of the subgame that starts at a time after the arrival of the fourth technology depends on the realization of T_4 and thus on the realization of τ_3. It turns out that there are five important intervals. This implies that there are four critical values for τ_3, denoted by $\tau_3^* (i)$, $i \in \{1, 2, 3, 4\}$. On each of these intervals the configuration of the figure in which L, F and M are depicted is the same.

A.1.1. $\tau_3 \in [0, \tau_3^* (1)) = [0, 0.800591)$

In Figure 2 the three curves are plotted for $\tau_3 = 0.5$.

Here technology 4 is invented just after the invention date of technology 3. Therefore,

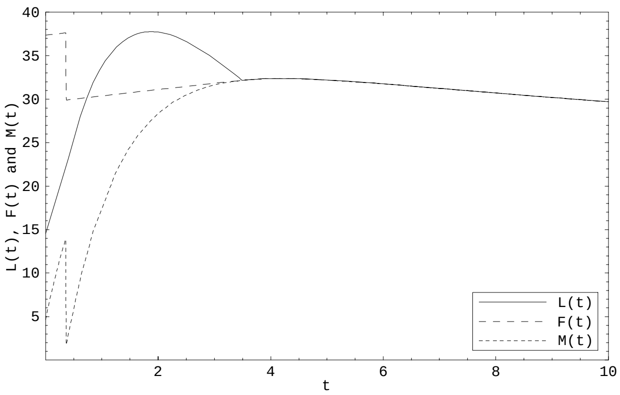

Figure 2. Leader, follower and joint moving curves for the subgame that starts at time T_4 if $\tau_3 = 0.5$.

technology 4 dominates technology 3 very quickly. At time $t_{34}^{L}(\tau_3)$ the leader is indifferent between adopting technology 3 and technology 4:

$$V\left(t_{34}^{L}(\tau_3), 3, w_4^F(3), 4\right) = V\left(t_{34}^{L}(\tau_3), 4, w_4^F(4), 4\right).$$

Note that the follower is not indifferent, because the follower curve jumps down at time t_{34}^{L}. There are two equilibria with symmetric strategies. Define the preemption time t_4^P as

$$t_4^P = \min\{t | L(t) = F(t)\} = 0.734579.$$

For $t > t_4^P$ it holds that $L(t) > F(t) > M(t)$. Therefore, the game that starts at time t_4^P is a preemption game.

At $T > t_4^P$ it is in the interest of each firm to adopt technology 4 right away (since $L(T) > F(T)$). But if a firm knows that the other will adopt at this particular time, it wants to preempt at $T - \varepsilon$. Reasoning backwards, at any t between t_4^P and T, firms want to preempt to avoid being preempted later on. As shown in Appendix B this leads to the following equilibrium: with probability one-half a firm becomes leader and adopts technology

4 at time $T_4 + t_4^P$, where $t_4^P = 0.734579$. The other firm is follower and adopts technology 4 at time $T_4 + w_4^F(4)$, where $w_4^F(4) = 3.49080$. With probability one-half the roles are reversed. We conclude that the game ends for sure at time t_4^P. The probability of a mistake, i.e. both firms adopting technology 4 at time t_4^P leaving them with a low payoff $M\left(t_4^P\right) < F\left(t_4^P\right)$, is equal to zero (see Appendix B). Both firm's values are equal, i.e. there is rent-equalization. A firm's value (discounted to time T_4) equals $\frac{1}{2}F\left(t_4^P\right) + \frac{1}{2}L\left(t_4^P\right) = 30.1722$.

At $T < t_4^P$, it holds that $F(T) > L(T)$ and $L'(T) > 0$. Therefore, both firms wait until t_4^P where the above described preemption game starts.

The boundary $\tau_3^*(1)$ is derived by solving the equation $t_{34}^L(\tau_3^*(1)) = t_4^P$. Thus if $\tau_3 = \tau_3^*(1)$ the leader is indifferent between two strategies exactly at the preemption time t_4^P.

A.1.2. $\tau_3 \in [\tau_3^*(1), \tau_3^*(2)) = [0.800591, 1.17938)$

The leader, follower and joint-moving curves are plotted in Figure 3 for $\tau_3 = 1$.

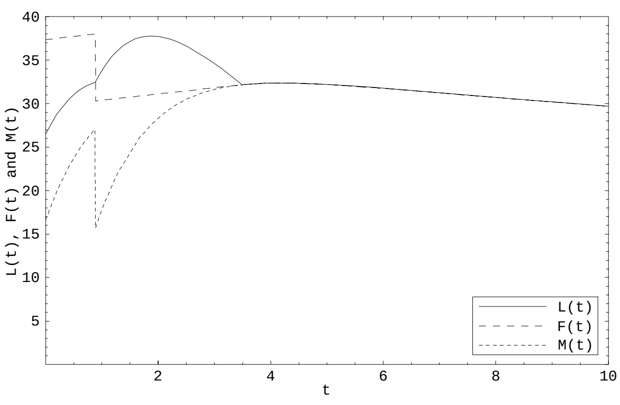

Figure 3. Leader, follower and joint moving curves for the subgame that starts at time T_4 if $\tau_3 = 1$.

In this τ_3-region there is no equilibrium with symmetric strategies. Here, there are four equilibria for this subgame. At time $T_4 + t_{34}^L(\tau_3)$ the leader is indifferent between adopting technology 3 and adopting technology 4:

$$V\left(t_{34}^L(\tau_3), 3, w_4^F(3), 4\right) = V\left(t_{34}^L(\tau_3), 4, w_4^F(4), 4\right).$$

The subgame that starts at time $t > T_4 + t_{34}^L(\tau_3)$ is a preemption game and in equilibrium the expected value for both firms is $V\left(w_4^F(4), 4, t, 4\right)$, i.e. the follower value if the leader adopts technology 4 at time $T_4 + t$ and the follower adopts technology 4 at time $T_4 + w_4^F(4)$. This subgame ends at time $T_4 + t$ with probability one.

Adopting before time $T_4 + t_{34}^L(\tau_3)$ is not optimal for a firm, because the follower value is larger than the leader value and the leader value is increasing.

The story above implies that the game will end at time $T_4 + t_{34}^L(\tau_3)$ with probability one. The leader has two possible strategies: adopt technology 3 and adopt technology 4. The follower's optimal reply always is adopting technology 4. Thus there are two types of equilibria. In the first type the leader adopts technology 3 and the follower technology 4 and in the second type the leader and the follower both adopt technology 4. Right at $T_4 + t_{34}^L(\tau_3)$ the leader's value is equal in both equilibria, but the follower's value is larger in the equilibrium where the leader adopts technology 3. In other words, the equilibrium in which the leader adopts technology 3 Pareto dominates the other equilibrium and that is why we use this equilibrium in further calculations. We assume that nature assigns to a firm the role of leader and that both firms have equal probability of being assigned leader.

The expected value of each firm equals

$$\frac{1}{2}\left(V\left(t_{34}^{L}\left(\tau_{3}\right),3,w_{4}^{F}\left(3\right),4\right)+V\left(w_{4}^{F}\left(3\right),4,t_{34}^{L}\left(\tau_{3}\right),3\right)\right),$$

where $w_4^F(3) = 3.26767$.

A.1.3. $\tau_3 \in [\tau_3^*(2), \tau_3^*(3)] = [1.17938, 1.87931]$

On this interval the leader curve is decreasing on the interval $\left(T_4 + S_{34}^L(\tau_3), T_4 + t_{34}^L(\tau_3)\right)$, where

$$S_{34}^{L}\left(\tau_{3}\right):=\arg\max_{t\in\left[0,t_{34}^{L}\left(\tau_{3}\right)\right]}V\left(t,3,w_{4}^{F}\left(3\right),4\right).$$

The boundary $\tau_3^*(2)$ is the solution of the equation $S_{34}^L(\tau_3^*(2)) = t_{34}^L(\tau_3^*(2))$. In Figure 4 the three curves are plotted for $\tau_3 = 1.5$.

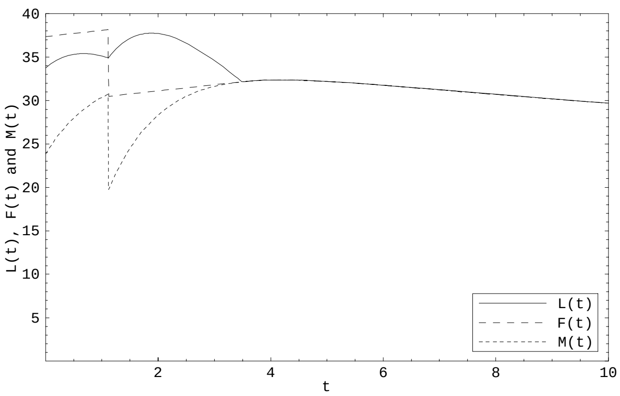

Figure 4. Leader, follower and joint moving curves for the subgame that starts at time T_4 if $\tau_3 = 1.5$.

Since the follower curve lies above the leader curve, we have a war of attrition on this interval. Since

$$V\left(t,3,w_{4}^{F}\left(3\right),4\right)>V\left(w_{4}^{F}\left(4\right),4,t_{34}^{L}\left(\tau_{3}\right),4\right),$$

there does not exist an equilibrium with symmetric strategies for this game (cf. Hendricks *et al.* (1988)). There are two equilibria for this game. In each, the leader adopts technology 3 at time $T_4+S_{34}^L(\tau_3)$ and the follower adopts technology 4 at time $T_4+w_4^F(3)$. As before we assume that nature assigns a firm to be leader or follower. Both firms have equal probability of being assigned leader. Thus the expected value of a firm equals

$$\frac{1}{2}\left(V\left(S_{34}^L(\tau_3),3,w_4^F(3),4\right)+V\left(w_4^F(3),4,S_{34}^L(\tau_3),3\right)\right).$$

The subgames that start at time $t>T_4+t_{34}^L(\tau_3)$ have not changed.

A.1.4. $\tau_3\in(\tau_3^*(3),\tau_3^*(4))=(1.87931,1.89322)$

In these subgames the value of the leader exceeds the value of the follower in a subinterval of the interval $\left(0,t_{34}^L(\tau_3)\right)$ (see Figure 5).

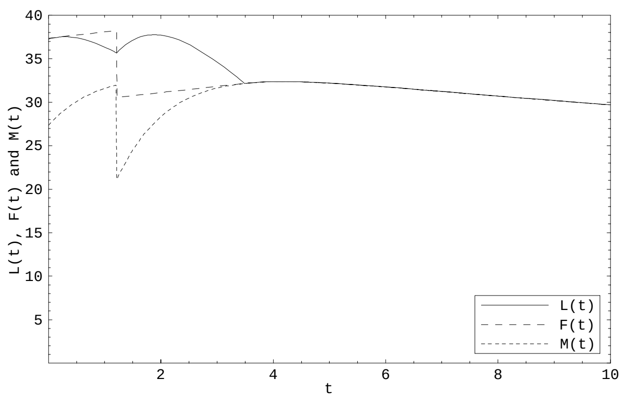

Figure 5. Leader, follower and joint moving curves for the subgame that starts at time T_4 if $\tau_3=1.88$.

These subgames are preemption games. There are two equilibria with symmetric strategies. With probability one-half a firm becomes leader and adopts technology 3 at time $T_4 + t_{34}^P(\tau_3)$, where $t_{34}^P(\tau_3)$ is defined by

$$t_{34}^P(\tau_3) := \min\left\{t \left| V\left(t, 3, w_4^F(3), 4\right) = V\left(w_4^F(3), 4, t, 3\right)\right.\right\}.$$

The other firm is follower and adopts technology 4 at time $T_4 + w_4^F(3)$, and with probability one-half the roles are reversed. According to Appendix B, due to rent equalization, there is zero probability of mistake, i.e. both firms adopting technology 3 at time $T_4 + t_{34}^P(\tau_3)$. Both firm's values are equal so that there is rent-equalization. The firm's value (discounted to time T_4) equals

$$V\left(t_{34}^P(\tau_3), 3, w_4^F(3), 4\right).$$

The boundary $\tau_3^*(3)$ is defined as the smallest τ_3 for which there exists an $t_{34}^P(\tau_3)$.

A.1.5. $\tau_3 \in [\tau_3^*(4), \infty) = [1.89322, \infty)$

The boundary $\tau_3^*(4)$ is defined as the smallest τ_3 for which the preemption time $t_{34}^P(\tau_3)$ equals 0. Thus, in this region the games end at time T_4 with probability one. The leader's value at time T_4 exceeds the follower's value at time T_4 and that is why there is a positive probability of a mistake (see Figure 6).

Define (see Appendix B)

$$\alpha(t|\tau_3) = \frac{V\left(t, 3, w_4^F(3), 4\right) - V\left(w_4^F(3), 4, t, 3\right)}{V\left(t, 3, w_4^F(3), 4\right) - V(t, 3, t, 3)}.$$

The probability of a firm to become leader (adopt technology 3 at time T_4) or to become follower (adopt technology 4 at time $T_4 + w_4^F(3)$) equals

$$\frac{1 - \alpha(0|\tau_3)}{2 - \alpha(0|\tau_3)},$$

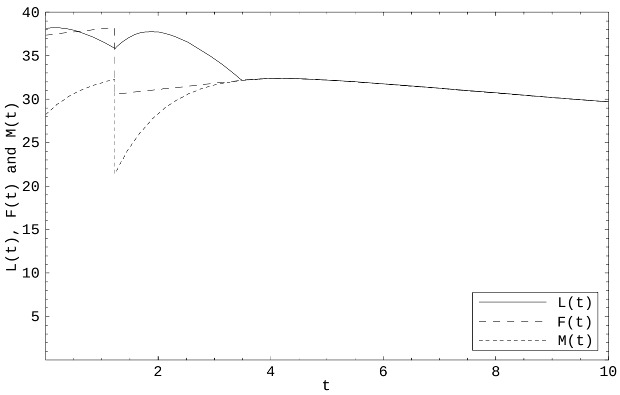

Figure 6. Leader, follower and joint moving curves for the subgame that starts at time T_4 if $\tau_3 = 2$.

and the probability of a mistake (both firms adopting technology 3 at time T_4) equals

$$\frac{\alpha(0\,|\tau_3)}{2-\alpha(0\,|\tau_3)}.$$

Using these probabilities, it is not hard to derive that the expected value of the firm equals

$$V\left(w_4^F(3), 4, 0, 3\right).$$

A.1.6. Summary

The expected payoff of the game equals

$$\Omega_4(\tau_3) = \begin{cases} V\left(w_4^F(4), 4, t_4^P, 4\right) & \tau_3 \in [0, \tau_3^*(1)) \\ \frac{1}{2}\left(V\left(t_{34}^L(\tau_3), 3, w_4^F(3), 4\right) + V\left(w_4^F(3), 4, t_{34}^L(\tau_3), 3\right)\right) & \tau_3 \in [\tau_3^*(1), \tau_3^*(2)) \\ \frac{1}{2}\left(V\left(S_{34}^L(\tau_3), 3, w_4^F(3), 4\right) + V\left(w_4^F(3), 4, S_{34}^L(\tau_3), 3\right)\right) & \tau_3 \in [\tau_3^*(2), \tau_3^*(3)] \\ V\left(t_{34}^P(\tau_3), 3, w_4^F(3), 4\right) & \tau_3 \in (\tau_3^*(3), \tau_3^*(4)) \\ V\left(w_4^F(3), 4, 0, 3\right) & \tau_3 \in [\tau_3^*(4), \infty) \end{cases}.$$

A.2. Game Starting at Time $[T_3, T_4)$

Using the expression for $\Omega_4(\tau_3)$ we derive the waiting curve for the subgames starting at some time $t \in [T_3, T_4)$:

$$W(t) = \frac{\pi(\theta_0,\theta_0)}{r}\left(1 - e^{-rt}\right)$$
$$+ e^{-rt} \int\limits_{u_3=0}^{\infty} \left[\int\limits_{v_3=0}^{u_3} \pi(\theta_0,\theta_0)\, e^{-rv_3} dv_3 + e^{-ru_3}\Omega_4(u_3) \right] \lambda e^{-\lambda u_3} du_3.$$

The equilibria in this subgame depend on τ_2. There are four important intervals, thus there are three critical values for τ_2: $\tau_2^*(i)$, $i \in \{1,2,3\}$. For the moment we derive the equilibria in the case that technology 4 has not arrived yet.

A.2.1. $\tau_2 \in [0, \tau_2^*(1)) = [0, 1.24843)$

In Figure 7 the leader, follower, joint moving and waiting curves are plotted for $\tau_2 = 1$.

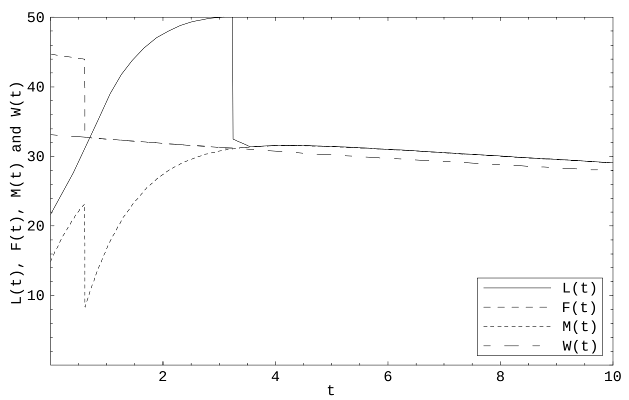

Figure 7. Leader, follower, joint moving and waiting curve for the subgame that starts at time T_3 if $\tau_2 = 1$.

It turns out that waiting is not an option, because the leader curve exceeds the waiting curve for some points in time. These games are preemption games and there are two equilibria with symmetric strategies. With probability one-half a firm becomes leader and adopts technology 3 at time $T_3 + t_{34}^P$, where $t_{34}^P = 0.727495$. The other firm is follower and is expected to adopt technology 4 at time $T_4 + w_4^F(3)$, and with probability one-half the roles are reversed. There is zero probability of mistake, i.e. both firms adopting technology 3 at time $T_3 + t_{34}^P$. Both firm's values are equal so that there is rent-equalization. The firm's value (discounted to time T_3) equals 32.6639.

If $\tau_2 = \tau_2^*(1) = 1.24843$ it holds that $t_{23}^L(\tau_2) = t_{34}^P$.

A.2.2. $\tau_2 \in [\tau_2^*(1), \tau_2^*(2)) = [1.24843, 2.94586)$

In this region there are two types of equilibria, but none of them is supported by symmetric strategies. In Figure 8 the four curves are plotted for $\tau_2 = 2$.

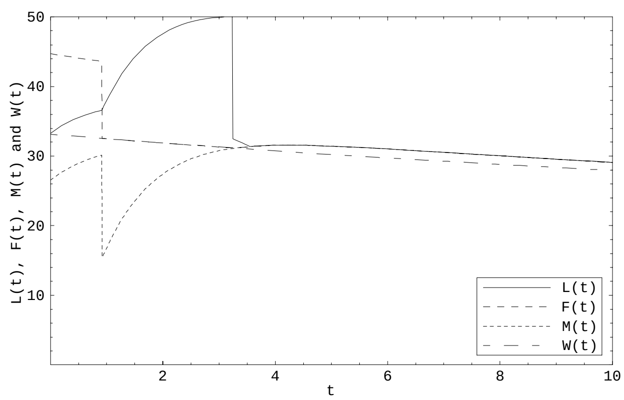

Figure 8. Leader, follower, joint moving and waiting curve for the subgame that starts at time T_3 if $\tau_2 = 2$.

We use the following type in further calculations. The leader adopts technology 2 at time

$T_3 + t_{23}^L(\tau_2)$ and the follower is expected to wait for technology 4 and adopt it at time $T_4 + w_4^F(2)$, where $w_4^F(2) = 2.97998$. Nature assigns the roles to the firms. The expected value of the firms is equal to

$$\frac{1}{2}\left(V\left(t_{23}^L(\tau_2), 2, w_4^F(2), 4\right) + V\left(w_4^F(2), 4, t_{23}^L(\tau_2), 2\right)\right).$$

The second boundary, $\tau_2^*(2) = 2.94586$, is derived by solving the following equation

$$t_{23}^L(\tau_2) = S_{24}^L(\tau_2),$$

where

$$S_{24}^L(\tau_2) := \arg \max_{t \in \left[0, t_{24}^L(\tau_2)\right]} V\left(t, 2, w_4^F(2), 4\right).$$

A.2.3. $\tau_2 \in [\tau_2^*(2), \tau_2^*(3)) = [2.94586, 3.95758)$

Again no equilibrium with symmetric strategies in this region exists (see Figure 9 for a plot of the curves in this region).

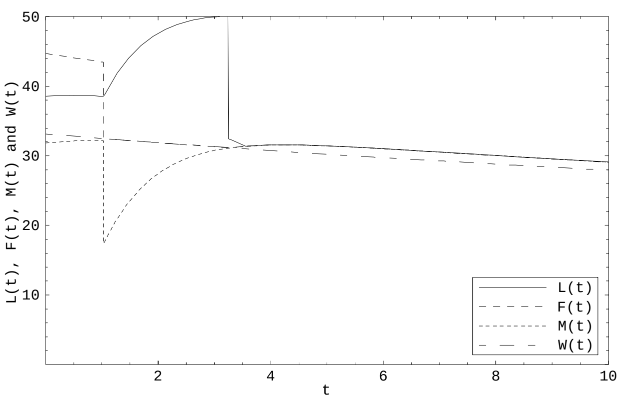

Figure 9. Leader, follower, joint moving and waiting curve for the subgame that starts at time T_3 if $\tau_2 = 3.5$.

In equilibrium the leader adopts technology 2 at time $T_3 + S_{24}^L(\tau_2)$ and the follower is expected to adopt technology 4 at time $T_4 + w_4^F(2)$. As before the roles are assigned by nature. The firm's expected value equals

$$\frac{1}{2}\left(V\left(S_{24}^L(\tau_2), 2, w_4^F(2), 4\right) + V\left(w_4^F(2), 4, S_{24}^L(\tau_2), 2\right)\right).$$

The critical value $\tau_2^*(3)$ $(= 3.95758)$ is defined by

$$t_2^*(3) := \min\left\{\tau_2 | \, S_{24}^L(\tau_2) = 0\right\}.$$

A.2.4. $\tau_2 \in [\tau_2^*(3), \infty) = [3.95758, \infty)$

In Figure 10 the leader, follower, joint moving and waiting curves are plotted for $\tau_2 = 4$.

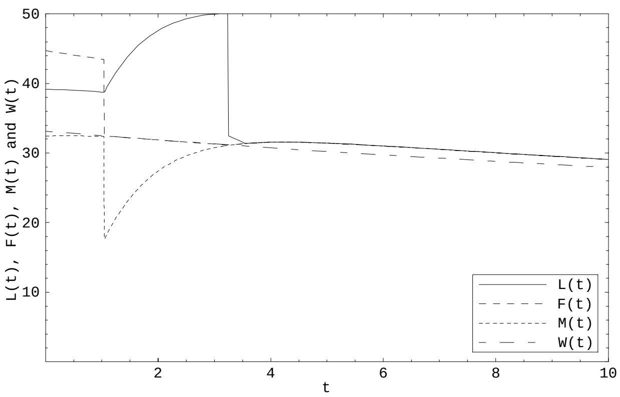

Figure 10. Leader, follower, joint moving and waiting curve for the subgame that starts at time T_3 if $\tau_2 = 4$.

The leader adopts technology 2 at time T_3 and the follower is expected to adopt technology 4 at time $T_4 + w_4^F(2)$ in equilibrium. Roles are assigned by nature. Expected firm values are given by

$$\frac{1}{2}\left(V\left(0,2,w_4^F\left(2\right),4\right)+V\left(w_4^F\left(2\right),4,0,2\right)\right).$$

A.2.5. Summary

The expected payoff of the game equals

$$\Omega_3\left(\tau_2\right)=\begin{cases}
\begin{array}{l}\int\limits_{\tau_3=0}^{t_{34}^P}\left[\int\limits_{v=0}^{\tau_3}\pi\left(\theta_0,\theta_0\right)e^{-rv}dv\right.\\
\left.+e^{-r\tau_3}\Omega_4\left(\tau_3\right)\right]\lambda e^{-\lambda\tau_3}d\tau_3\\
+\int\limits_{\tau_3=t_{34}^P}^{\infty}V\left(w_4^F\left(3\right),4,t_{34}^P,3\right)\lambda e^{-\lambda\tau_3}d\tau_3\end{array} & \tau_2\in\left[0,\tau_2^*\left(1\right)\right)\\
\begin{array}{l}\int\limits_{\tau_3=0}^{t_{23}^L(\tau_2)}\left[\int\limits_{v=0}^{\tau_3}\pi\left(\theta_0,\theta_0\right)e^{-rv}dv\right.\\
\left.+e^{-r\tau_3}\Omega_4\left(\tau_3\right)\right]\lambda e^{-\lambda\tau_3}d\tau_3\\
+\int\limits_{\tau_3=t_{23}^L(\tau_2)}^{\infty}\frac{1}{2}\left(V\left(t_{23}^L\left(\tau_2\right),2,w_4^F\left(2\right),4\right)\right.\\
\left.+V\left(w_4^F\left(2\right),4,t_{23}^L\left(\tau_2\right),2\right)\right)\lambda e^{-\lambda\tau_3}d\tau_3\end{array} & \tau_2\in\left[\tau_2^*\left(1\right),\tau_2^*\left(2\right)\right)\\
\begin{array}{l}\int\limits_{\tau_3=0}^{S_{24}^L(\tau_2)}\left[\int\limits_{v=0}^{\tau_3}\pi\left(\theta_0,\theta_0\right)e^{-rv}dv\right.\\
\left.+e^{-r\tau_3}\Omega_4\left(\tau_3\right)\right]\lambda e^{-\lambda\tau_3}d\tau_3\\
+\int\limits_{\tau_3=S_{24}^L(\tau_2)}^{\infty}\frac{1}{2}\left(V\left(S_{24}^L\left(\tau_2\right),2,w_4^F\left(2\right),4\right)\right.\\
\left.+V\left(w_4^F\left(2\right),4,S_{24}^L\left(\tau_2\right),2\right)\right)\lambda e^{-\lambda\tau_3}d\tau_3\end{array} & \tau_2\in\left[\tau_2^*\left(2\right),\tau_2^*\left(3\right)\right)\\
\frac{1}{2}\left(V\left(0,2,w_4^F\left(2\right),4\right)+V\left(w_4^F\left(2\right),4,0,2\right)\right) & \tau_2\in\left[\tau_2^*\left(3\right),\infty\right)
\end{cases}.$$

A.3. Games Starting at Time $t \in [T_2, T_3)$

Using the expression for $\Omega_3(\tau_2)$ we derive the waiting curve for the subgames starting at some time $t \in [T_2, T_3)$:

$$W(t) = \frac{\pi(\theta_0, \theta_0)}{r}\left(1 - e^{-rt}\right) + e^{-rt} \int_{u_2=0}^{\infty} \left[\int_{v_2=0}^{u_2} \pi(\theta_0, \theta_0) e^{-rv_2} dv_2 + e^{-ru_2} \Omega_3(u_2) \right] \lambda e^{-\lambda u_2} du_2.$$

It turns out that the equilibria in this subgames do not depend on τ_1. This can be explained by the fact that $t_{12}^L(\tau_1) < t_{24}^P$ for all τ_1.

A.3.1. $\tau_1 \in [0, \infty)$

In Figure 11 the four curves are plotted for $\tau_1 = 2$.

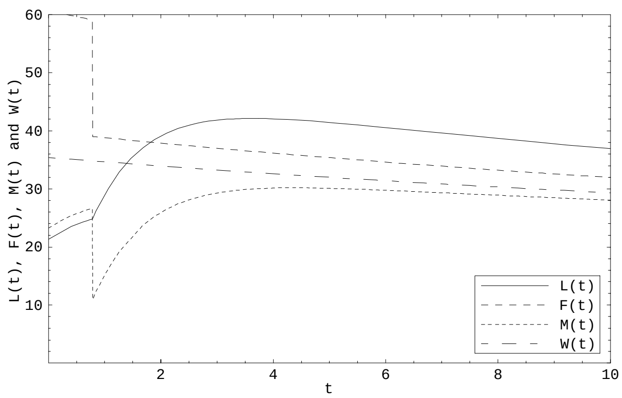

Figure 11. Leader, follower, joint moving and waiting curve for the subgame that starts at time T_2 if $\tau_1 = 2$.

There are two equilibria, where each occurs with a probability one-half. The leader adopts technology 2 at time $T_2 + t_{24}^P$, where $t_{24}^P = 1.81706$, and the follower is expected to adopt technology 4 at time $T_4 + w_4^F(2)$. The firm's expected value equals 38.0414.

A.3.2. Summary

The expected payoff of the game equals

$$\Omega_2(\tau_1) = \begin{cases} \int\limits_{u_2=0}^{t_{24}^P} \left[\int\limits_{v_2=0}^{u_2} \pi(\theta_0, \theta_0) e^{-rv_2} dv_2 \right. \\ \left. \quad + e^{-ru_2} \Omega_3(u_2) \right] \lambda e^{-\lambda u_2} du_2 & \tau_1 \in [0, \infty) . \\ + \int\limits_{u_2=t_{24}^P}^{\infty} V\left(w_4^F(2), 4, t_{24}^P, 2\right) \lambda e^{-\lambda u_2} du_2 \end{cases}$$

B. Preemption Games

In this appendix we give on overview of the mathematics that are needed to derive the equilibria of the preemption games that appear in Subsections 3.2-3.4. Since we study symmetric firms we restrict ourselves to symmetric strategies.

Consider a preemption game with time horizon $[0, \infty)$[10] and the following characteristics:

$$L(0) \geq F(0) > M(0), \tag{34}$$

$$L(t) > F(t) > M(t), \quad t \in (0, \infty), \tag{35}$$

$$\lim_{t \to \infty} L(t) = \lim_{t \to \infty} F(t) = \lim_{t \to \infty} M(t). \tag{36}$$

Let us consider a subgame that starts at time $t > 0$. The payoff scheme (34)-(36) leads to the following two observations: (1) both firms want to become the leader at time t,

[10] Note that the preemption games of Subsections 3.2-3.4 can be fitted into this time horizon with a time transformation.

since $L(t) > F(t)$ and (2) both firms do not like simultaneous adoption at time t, since $M(t) < F(t)$. Thus it is attractive to be the first investor, but when both firms apply the strategy "invest at time t" with probability one, they both end up with the low joint adoption payoff $M(t)$.

Fudenberg and Tirole (1985) argued that a strategy in this kind of games can not be represented by a single distribution function. It is necessary to be able to distinguish between types of atoms. Therefore the strategy of a player consists of two functions $(G(t), \alpha(t))$. $G(t)$ is the probability that one firm has adopted by some time t given that the other has not adopted. The function $\alpha(t)$ measures the intensity of atoms in the interval $[t, t+dt]$. By definition (see Fudenberg and Tirole (1985)) a positive $\alpha(t)$ implies that a player is sure to adopt by time t, i.e. $G(t) = 1$.

Next we give an interpretation of the $\alpha(t)$ function. Assume that the players both use the same strategy and let τ be the smallest point in time at which $\alpha(t)$ is positive: $\tau = \inf\{t | \alpha(t) > 0\}$. We know for sure that the game will be stopped by time τ. This can be seen as a game that is repeated (if necessary infinitely often) in the time interval $[\tau, \tau + \varepsilon)$ with ε small, until one of the players has stopped for sure. In each stage game a player stops with probability $\alpha(\tau)$.

Thus the probability that a player stops the game at time τ, $\Pr(\text{one})$, equals

$$\begin{aligned}
\Pr(\text{one}) &= \alpha(\tau)(1-\alpha(\tau)) + (1-\alpha(\tau))(1-\alpha(\tau))\Pr(\text{one}), \\
P(\text{one}) &= \frac{1-\alpha(\tau)}{2-\alpha(\tau)}, \qquad (37)
\end{aligned}$$

and the probability that both players stop the game at τ, $\Pr(\text{two})$, equals

$$\Pr(\text{two}) = \alpha(\tau)\,\alpha(\tau) + (1-\alpha(\tau))\,(1-\alpha(\tau))\Pr(\text{two}),$$
$$\Pr(\text{two}) = \frac{\alpha(\tau)}{2-\alpha(\tau)}. \tag{38}$$

Thus each player stops the game itself with probability $\frac{1-\alpha(t)}{2-\alpha(t)}$ and with probability $\frac{\alpha(t)}{2-\alpha(t)}$ both players stop the game.

If $\alpha(\tau) = 0$ the probabilities are equal to their first-order Taylor expansions:

$$\Pr(\text{one}) = \frac{1}{2}, \tag{39}$$
$$\Pr(\text{two}) = 0. \tag{40}$$

Back to the subgame that starts at time $t > 0$. Given that the other firm has not invested, it is optimal for a firm to invest at time t, thus

$$G(s) = 1, \quad s \geq t. \tag{41}$$

This implies that we know for sure that the game ends at time t. Let $\alpha_i(t)$ be the $\alpha(t)$ function of firm i. Then, while suppressing the time arguments, the value, V_1, of firm 1 under these strategies equals

$$V_1 = \alpha_1(1-\alpha_2)\,L + (1-\alpha_1)\,\alpha_2 F + \alpha_1\alpha_2 M + (1-\alpha_1)(1-\alpha_2)\,V_1. \tag{42}$$

This equation reflects that with probability $\alpha_1(1-\alpha_2)$ firm 1 invests first so that it obtains payoff L, with probability $(1-\alpha_1)\,\alpha_2$ firm 2 invests as first so that firm 1 gets the follower payoff, with probability $\alpha_1\alpha_2$ the firms invest at the same time leading to a payoff equal to

M, while with probability $(1-\alpha_1)(1-\alpha_2)$ nothing has happened and the game is repeated. Rewriting (42) gives

$$V_1 = \frac{\alpha_1 (1-\alpha_2) L + (1-\alpha_1)\alpha_2 F + \alpha_1\alpha_2 M}{1-(1-\alpha_1)(1-\alpha_2)}. \tag{43}$$

To find the optimal value for α_1 we differentiate (43) with respect to α_1 and put this expression equal to zero. This eventually leads to the following equality:

$$\frac{\partial V_1}{\partial \alpha_1} = \frac{\alpha_2 \left((1-\alpha_2) L - F + \alpha_2 M\right)}{\left(1-(1-\alpha_1)(1-\alpha_2)\right)^2} = 0. \tag{44}$$

It is easy verified that $\frac{\partial^2 V_1}{\partial \alpha_1^2} < 0$, so that satisfying (44) indeed leads to a maximum value of the firm. Since we only consider symmetric strategies we impose that

$$\alpha_1(t) = \alpha_2(t) = \alpha(t). \tag{45}$$

Combining (44) and (45) leads to the following optimal value for $\alpha(t)$:[11]

$$\alpha(t) = \frac{L(t) - F(t)}{L(t) - M(t)}. \tag{46}$$

From equations (34)-(36) and (46) we derive that $\alpha(t) \in [0,1]$. Substituting (46) and (45) in (43) gives $V_1(t) = F(t)$. Thus the expected value for each firm is the follower value. For a formal proof that (41) together with (46) is a perfect equilibrium we refer to Fudenberg and Tirole (1985).

Equation (46) also implies that in case of rent equalization $(L(t) = F(t))$, $\alpha(t) = 0$. Then (39) and (40) apply, so that it immediately follows that the probability of a mistake (i.e. both firms investing at the same point of time) equals zero.

[11] A similar value was found in Fudenberg and Tirole (1985) and Simon (1987).

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